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Cosmological constant problems and the renormalization group

Ilya L Shapiro[1,3] **and Joan Solà**[2,4]

[1] Departamento de Física—ICE, Universidade Federal de Juiz de Fora, MG, Brazil
[2] Dep. ECM Universitat de Barcelona, and CER for Astrophysics, Particle Physics and Cosmology, Diagonal 647, Barcelona, Catalonia, Spain

E-mail: shapiro@fisica.ufjf.br and sola@ifae.es

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Abstract
The cosmological constant problem emerges when quantum field theory is applied to the gravitational theory, due to the enormous magnitude of the induced energy of the vacuum. The unique known solution of this problem involves an extremely precise fine tuning of the vacuum counterpart. We review a few of the existing approaches to this problem based on the account of the quantum (loop) effects and pay special attention to those involving the renormalization group.

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1. Introduction

There are two main reasons for introducing the cosmological constant, both theoretical and experimental. The theoretical reason is that there are many distinct sources of the cosmological constant (CC) in quantum field theory (QFT) and particle physics. Below we shall discuss some of them. The experimental/observational evidence of an accelerated expansion of the universe comes from the type-Ia supernova observations [1], from the CMB data [2] and also from the available rich information about the galaxy distribution [3]. Usually, the nonzero vacuum energy is referred to as dark energy (DE), because it does interact with the matter content of the universe only gravitationally and its ultimate nature is unknown. There are many candidates to play the role of DE (e.g. quintessence or phantom energy [4, 5]), but due to the mentioned theoretical arguments the CC is the main candidate for the role of DE. However, the situation is spoiled by the CC problems, including the problem of fine tuning [6] and the related coincidence problem [7]. One can easily identify these problems as a hierarchy problems arising due to the enormous scale difference between large-scale gravitational

[3] Also at Tomsk State Pedagogical University, Tomsk, Russia.
[4] Associated with Instituto de Ciencias del Espacio-CSIC.

physics (cosmology) and the short-scale characteristic of high-energy particle physics—see [8] for a detailed discussion. In this situation, even very small quantum corrections to the CC may be relevant and, in principle, lead to a certain observable consequences. Indeed, the renormalization group (RG) is the most economic way to parametrize these quantum corrections and investigate their phenomenological impact. The purpose of this paper is to put together a few different approaches to the CC problem based on the renormalization group. The paper is mainly based on the original papers [8–12].

2. Why do we need CC in QFT?

There are two sources of the vacuum energy which have essentially distinct origin. One of them is the classical gravitational action. Besides the Einstein–Hilbert term, this action can include additional terms, both local and non-local. The simplest of them is the CC. If we consider the quantum theory of matter fields on the classical gravitational background, the consistency requirement is that the action of vacuum has the form (see, e.g., [13, 14]) $S_{\text{vac}} = S_{\text{EH}} + S_{\text{HD}}$, where

$$S_{\text{EH}} = -\int \mathrm{d}^4x\sqrt{-g}\left\{\frac{1}{16\pi G_{\text{vac}}}R + \Lambda_{\text{vac}}\right\}, \tag{1}$$

and S_{HD} include fourth derivative terms[5]. Without any of the mentioned terms (in particular the Λ term), the theory is not renormalizable [15]. When looking at the typical cosmic scale energy, it seems that the UV divergences are irrelevant. The cosmic scale energy can be characterized by the Hubble parameter $H_0 \sim 10^{-42}$ GeV, which is about 30 orders of magnitude smaller than the mass m_ν of the lightest neutrino or than the energy of the relic radiation photon. However, the action of vacuum should be the same at all periods of the history of the universe, including the very early epoch when the Hubble parameter was much greater and UV divergences become a serious problem if the theory would be non-renormalizable.

The cosmological term is thus an unavoidable element of the vacuum action (1), meaning that if the DE is modelled in alternative ways (e.g. through quintessence) the contribution from the CC term is always there and must necessarily be taken into account [8]. The natural question is what is the natural magnitude of the vacuum CC. In order to address this problem, let us remember the renormalization group equation for the CC,

$$\mu\frac{\mathrm{d}\Lambda_{\text{vac}}}{\mathrm{d}\mu} = \beta_\Lambda = \frac{m_s^4}{2} - 2m_f^4, \tag{2}$$

where we just took into account the contributions of a massive scalar and a fermion. μ is the typical energy of the external field (graviton, in the case). The last expression shows that the natural range of Λ_{vac} is given by the fourth power of the mass of the heaviest particle. Obviously, this will produce a serious conflict with the measured value of the CC, unless the only contribution would be from neutrinos of $m_\nu \sim 10^{-3}$ eV [10]. In order to reduce this estimate we need a cancellation between bosons and fermions in equation (2). Supersymmetry (SUSY) [16] may, therefore, help to reduce the minimal admissible value of Λ_{vac}. However, at low energies SUSY is known to be broken, because the proper model for the physics up to the Fermi scale is the minimal standard model (SM) of particle physics. Hence, the RG equation (2) can only be applied for the energies comparable to the typical scale of the SM and, in order to be compatible with the high-energy running, the magnitude of the vacuum CC density should be of the order of the fourth power of the electroweak scale, namely $\sim 10^8$ GeV^4.

[5] We denote by Λ the CC density itself, sometimes indicated as ρ_V in the literature.

Another source of the CC is the induced action of gravity [17], e.g. the one which emerges through the electroweak (EW) spontaneous symmetry breaking (SSB). In the ground state of the Higgs potential of the SM, $V(\phi) = -(m^2/2)\phi^2 + (f/8)\phi^4$, the induced CC is

$$\Lambda_{\text{ind}} = \langle V \rangle = -\frac{m^4}{2f} = -\frac{1}{8} M_H^2 v^2 \approx -10^8 \text{ GeV}^4, \tag{3}$$

where M_H is the value of the Higgs boson mass and $v \simeq 250$ GeV is the VEV of the Higgs potential. Since we know from LEP experiments that $M_H > 114$ GeV we obtain the estimate (3). It is remarkable that this estimate coincides with the natural value of the vacuum parameter Λ_{vac}. However, what we really observe is the sum

$$\Lambda_{\text{obs}} = \Lambda_{\text{vac}} + \Lambda_{\text{ind}}, \tag{4}$$

where the induced quantity Λ_{ind} receives many different contributions (e.g. from the electroweak SSB, chiral symmetry breaking and other possible phase transitions, plus quantum corrections to these contributions) and cannot be, in principle, calculated exactly.

The real problem is that the cancellation of the two independent contributions Λ_{vac} and Λ_{ind} (even though they can be expected to be of the same order of magnitude, as we have seen above for the EW case) must be extremely precise. By virtue of the recent astronomical observations, the observed value of the CC density is

$$\Lambda_{\text{obs}} \approx 0.7\rho_c \sim 10^{-47} \text{ GeV}^4. \tag{5}$$

Looking at the situation from the QFT viewpoint, the cancellation between Λ_{ind} and Λ_{vac} can be provided by imposing a restriction on the independent parameter Λ_{vac}. This restriction is nothing but the renormalization condition and it should be implemented at the cosmic scale μ_c where the observations are performed:

$$\Lambda_{\text{vac}}(\mu_c) = \Lambda_{\text{obs}} - \Lambda_{\text{ind}}(\mu_c). \tag{6}$$

In the last relation, the second term on the rhs ($\Lambda_{\text{ind}}(\mu_c)$) is 55 orders of magnitude greater than the observable term. Therefore, the renormalization condition for Λ_{vac} at the scale $\mu = \mu_c$ reads as follows: the quantity $\Lambda_{\text{vac}}(\mu_c)$ must be equal to $-\Lambda_{\text{ind}}(\mu_c)$ up to the 55th digit, for otherwise we should never meet the small Λ_{obs} which is actually observed. If we assume that there was another phase SSB-based transition at a typical GUT scale ($M_X \sim 10^{16}$ GeV), there will be more than 110 orders of magnitude difference and, finally, the phase transition at the Planck scale would give 123 orders. The (extremely huge) problem of *why* the two terms cancel so accurately is the famous CC problem, the biggest conundrum ever (see [6] for a classical review).

3. CC problem is a hierarchy problem

The simplest option is just accepting this cancellation as a fact. One can compare the situation with the great success of the SM, which has plenty of phenomenological parameters like masses of the particles etc. Of course, we cannot measure any of them with the 55-order precision, but this only shows that our 'measurement' of the CC is extremely precise. The origin of this 'precision' is nothing but the difference between the MS Fermi scale where we evaluate the induced CC and the tiny cosmic scale where we observe the total CC. Hence, *the CC problem is a hierarchy problem*, which results from the conflict between the physics at different scales.

In fact, the CC case is even more complicated. Remember that the universe is not static and that the temperature of the relic radiation was much higher in the past than it is now. In the early universe, there was an epoch when this temperature was about the Fermi scale $T \propto M_F$.

At that temperature, according to the standard viewpoint, the symmetry in the potential gets restored and the induced CC disappears or just becomes many orders smaller. Thus, in the earlier universe the overall CC includes the vacuum contribution $\Lambda_{\rm vac}$ only. This means that our universe had been created from the very beginning such that $\Lambda_{\rm vac}$, after the symmetry breaking phase transition, should cancel the induced CC with the tremendous 55-order precision. This does not look natural at all, and we really have to worry about this.

In addition to the rapid changes of the CC due to the phase transition, there should be a renormalization group running of both induced and vacuum counterparts of the CC. This issue was discussed in detail in [8], see this reference for details. Here, we just note that the running involves, in one way or another, the masses of the constituents of the SM. It might happen that the contribution of some particle is suppressed because it shows up only at higher loops, but in any case all physical SM effects occur at scales whose ratio with the Fermi scale is far away from those 55 orders of magnitude associated with the CC problem. At this point, we can conclude that the CC problem is something fundamental and that its solution should perhaps also involve the explanation of the particles mass spectrum.

Below we shall give a brief review of some methods of solving the CC problem. There are many reviews (see, e.g., [18]), so we will refer the reader to this work for the list of existing approaches and mainly concentrate here on those which do not involve higher dimensions/branes and are related to the quantum effects.

4. Supersymmetry, strings and anthropic approach

There were many attempts to solve the CC problems introducing more symmetries, e.g. supersymmetry (SUSY), which simply forbids the contributions to the CC because the SUSY vacuum must have zero energy (see the discussion in [19, 20] on the subject). Unfortunately, from the general perspective SUSY does not look helpful in solving the CC problem. The reason is that SUSY is explicitly broken at the electroweak scale, and therefore it cannot prevent the vacuum energy from getting contributions similar to the SM ones, see (3). To put in another way, SUSY is a high-energy phenomenon. As a result, we find ourselves in the following situation: while at low energies SUSY is broken and this leaves no much hope to apply it for the solution of the CC problem, at high energies SUSY may effectively apply, but then it cannot solve the CC problem because this problem possibly does not exist anymore there. Indeed, at scales near M_P or above the full SUSY β-function of the Λ term (i.e. the full structure on the rhs of equation (2) at very high energies) is probably zero. Since there are no induced contributions at those energies, but only the vacuum term $\Lambda_{\rm vac}$, there are no fine tunings, and $\Lambda_{\rm vac}$ can naturally be of order M_P^4 and remain peacefully so around those energies simply because it does not run while $\beta_\Lambda = 0$.

Another hope is the (super)string theory. However, the choice of vacuum for the string theory is not unique, at least at the present-day state of knowledge. It might happen that the 'right' vacuum gives a 'correct' value of the CC [21]. At first sight it is unclear how this can affect the low-energy physics, for at low energies we have a very strong experimental confirmation that the appropriate theory is the SM and not the string theory. Be as it may, at the moment moving from QFT to string theory does not seem to help much, for after the process of compactification from 11 dimensions down to 3 we are left with a vastly complex 'landscape' consisting of some 10^{1000} metastable (non-supersymmetric) vacua where to entertain our choice of the ground state [22, 23]. There is an important new aspect, however. If we consider the existence of a very complicated vacuum structure of string theory together with the anthropic hypothesis, there is a hope to arrive at some consistent picture and maybe even learn some lessons about fundamental physics.

The anthropic approach is using the 'experimental' fact of our own existence. One can say it produced the main and in fact unique success concerning the CC problem. The evolution of density perturbations depends on the equation of state of the matter and vacuum content of the universe. The formation of and friendly conditions in galaxies and stellar systems impose rigid constraints on the evolution of density perturbations in the universe. In this way, in 1987 Weinberg predicted the positiveness of the CC [24]. More detailed considerations [25] have shown that the CC should be quite close to the astronomically observed value, for otherwise our world would not exist (see, however, different estimates and interpretations of different schemes of defining the weighting in calculating the anthropic probabilities of the universes with different values of CC in [26, 27]). Taken on its own the anthropic approach cannot answer the fundamental question of *why* we should exist at all. But from the landscape perspective the answer may be just an existence of many universes (a 'multiverse'), most of which had (or have) no chance to evaluate or are not visible for us.

5. Auto-relaxation or adjusting mechanisms

The measurement that the cosmological constant is non-vanishing is relatively recent, it is only from 1998. Prior to this date the general belief from the theoretical physics and cosmologist community was that the cosmological constant had to be exactly zero. Therefore, it was quite natural to seek for an efficient adjustment mechanisms in which the value of the vacuum energy relaxes to zero in a dynamical way. The prototype mechanism to achieve this dynamical adjustment was to use some scalar field under some suitable potential or motivated by some symmetry requirement (e.g. dilatation symmetry). There were a number of very interesting attempts to create a sort of automatic mechanism for relaxing the CC. The general idea is to consider a modified gravity theory, where the effect of cosmological constant is reduced. Typically, the original auto-relaxation models [28–30] involved a scalar field which moves to the minima of its potential along with the evolution of the universe. As an example, the 'cosmon' field introduced in [29] aimed at a Peccei–Quinn-like adjustment mechanism based on a dynamical selection of the vacuum state at zero VEV of the potential, $\langle V \rangle = 0$. More recently these ideas have been exploited profusely in various forms, such as the so-called quintessence scalar fields and the like [4], 'phantom' fields [5] etc, including some recently resurrected old ideas on adjusting mechanisms [31]. The main aim of these dynamical mechanisms is that the induced CC goes to zero automatically and the problem of a fine tuning between the induced and vacuum counterparts may be smoothed or banished. Of course, with the advent of the high precision cosmology experiments the hard job of the quintessence community is to understand why the relaxation point of the quintessence field is not precisely zero but some extremely small value of the potential! This value amounts to introduce a small mass for the quintessence field χ of order $m_\chi \sim H_0 \sim 10^{-42}$ GeV, which is some 17 orders of magnitude smaller than the upper bound on the photon mass from terrestrial experiments! Clearly, this shows the highly artificial character of the quintessence field from the particle physics standards. In the classical review by Weinberg [6] many of these dynamical approaches, and their presently insurmountable difficulties, are discussed in quite some detail. They all end up with some more or less obvious form of fine tuning.

6. Renormalization group and CC problem

The renormalization group (RG) is a conventional theoretical tool for investigating the scale dependence. As we have seen above, the CC problem is a violent conflict between two scales:

the high energy scale M (typically $M \sim M_P$) where $\Lambda_{\rm vac}$ and $\Lambda_{\rm ind}$ are defined and the (low energy) cosmic scale μ_c ($\ll M$) where we can observe their sum $\Lambda_{\rm obs}$. Therefore, it is quite natural to consider the CC problem from the RG perspective. Let us note that the idea of the renormalization group solution became quite popular in the last few years [32, 33]. The first practical realization has been suggested by Antoniadis and Mottola [34] in the framework of the quantum theory of conformal factor [35], which is a direct 4D analogue of the 2D Polyakov theory. Other realizations are based on different versions of the IR quantum gravity [36–38].

An alternative and perhaps the most simple way to achieve the IR screening of the CC has been suggested in [9]. This approach relies on the IR quantum effects of matter field rather than on the IR quantum gravity effects. In this section, we shall review the proposal of [9], trying to reformulate it in a slightly physical way. In the next section, we shall consider even more physical (and less ambitious) approach to the CC problem based on taking the quantum effects of matter fields into account.

Let us suppose that (i) the symmetry restoration at $T \propto M_F$ does not happen. Indeed, this means that the Higgs sector of the SM must be extended such that the non-restoration becomes possible. Then, we live not in the SM vacuum but in the GUT vacuum. (ii) The origin of all masses of the fields in the SM and beyond is the Coleman–Weinberg mechanism in some GUT model which describes the physics at a very high energy scale. The quantum symmetry breaking in the field Φ leads to the induced cosmological Λ and inverse Newton $1/G$ constants. Since all fields are massless, we do not need to introduce vacuum CC and G, and therefore no need to distinguish $\Lambda_{\rm obs}$ and $\Lambda_{\rm ind}$. The classical potential for the field Φ includes the nonminimal interaction term, since it is necessary for renormalizability $V = -\frac{\xi}{2}R\Phi^2 + f\Phi^4$. The induced gravitational quantities are

$$(16\pi G)^{-1} \sim \langle \xi(t)\Phi_0^2(t)\rangle, \qquad \Lambda \sim -\langle f(t)\Phi_0^4(t)\rangle. \tag{7}$$

In the last expressions we have introduced the dependence on the RG parameter $t = \ln(\mu/\mu_0)$. The explicit form of this dependence is a function of the GUT model under consideration. (iii) The fundamental theory has a huge number of copies $\mathcal{N}$ of all or some of its constituents. These fields couple to the scalar Φ in such a way that the β-function for the nonminimal parameter ξ has the form $\beta_\xi = \left(\xi - \frac{1}{6}\right) \cdot \tilde{A}(g^2)$, where $\tilde{A}(g^2) = -A\mathcal{N}g^2$ is a linear combination of the (gauge)^2, (Yukawa)^2 and scalar couplings. For simplicity, we assume that the square of Yukawa couplings and scalar couplings are proportional to the square of the gauge coupling g^2.

We require that A is positive such that the conformal fixed point $\xi = 1/6$ is stable in UV and the point $\xi = \infty$ to be an attractor in the IR low-energy limit (see general discussion and references to original papers in [14] and classification of the SU(N) and O(N) gauge models in [39]). (iv) For the sake of simplicity, we shall suppose that the gauge theory is finite—that is the β-functions for all coupling constants in the matter fields sector are zeros. This is not a necessary requirement, and one can consider another type of theories. For example, similar model has been recently discussed in [40], based on the quantum theory of scalar field with the coupling constant growing on in the IR limit.

In the leading log approximation for $V_{\rm eff}$ one can use the RG-improved classical potential V. In this way, we find

$$G^{-1} \propto \langle \xi(t)\Phi_0^2(t)\rangle \qquad \text{and} \qquad \frac{\Lambda}{G} \propto -\langle f(t)\Phi_0^4(t)\rangle. \tag{8}$$

Due to the gauge dependence, the RG for $\Phi(t)$ is not sufficient to determine asymptotic of this effective charge. At the same time we can easily find it using physical considerations. In fact, since $G(-\infty)$ have the finite classical value and $\xi(t) \sim \exp(-Ag^2t)$ we find that $\Phi(t) \sim \exp(Ag^2t)$. Now we take into account (8). Since in the finite models $\lambda(t) \equiv \lambda_0 = \mathrm{const}$, we find that in the IR limit $t \to -\infty$

$$\Lambda \propto \left\langle f(t)\Phi_0^4(t)\right\rangle \sim \exp(-2A\mathcal{N}g^2t) \qquad \text{and} \qquad \frac{\Lambda_{IR}}{\Lambda_{UV}} = \left(\frac{\mu_{IR}}{\mu_{UV}}\right)^{-2A\mathcal{N}g^2}, \tag{9}$$

where we assume CC running between the energy scales μ_{UV} and μ_{IR}

The result for the Λ_{IR} depends on the choice of the model (that defines A) as well as on the region of its application. Let us consider the running from the Planck scale $\mu_{UV} = M_P \approx 10^{19}$ GeV down to the present-day cosmic scale $\mu_{IR} = \mu_c \approx 10^{-42}$ GeV. The values of A have been calculated in a number of papers (see, for example, [14, 39]), and the typical values for A are between $1/(4\pi)^2$ and $50/(4\pi)^2$. For the maximal value we obtain

$$\Lambda_{IR}/\Lambda_{UV} \approx 10^{-6000\mathcal{N}g^2/(4\pi)^2}. \tag{10}$$

Taking $g \approx 10^{-1}$ and $\mathcal{N} = 1$, we can see that the value of Λ is decreasing on about four orders. But, if we take $\mathcal{N} = 30$ copies of the fields, we arrive at the tremendous 120 orders, which can solve the CC problem. The effect would be seen in the astronomic observations as a very slow decrease of the observable CC during the last few billion years.

The above model of IR screening for CC does not look like a natural solution of the CC problem. The need for the great number of copies of the fields (they must be massless, for otherwise they just decouple long before the IR limit), symmetry non-restoration at $T = M_F$ and all masses of the SM being the result of the dimensional transmutation at the GUT scale is not very appealing from the phenomenological point of view. The advantage of the model is that it is really free of a usual CC fine tuning.

7. Decoupling and cosmological constant

In this section, we shall consider the less ambitious program than in the previous ones. Namely, we will not try solving the great CC problem [6], neither the coincidence problem (see, however, the generalized RG model with cosmon field [43]). Instead we accept a purely phenomenological point of view and assume that the particle physics can be described by the known SM or some of its conventional extension. The question is whether it is possible that even in this situation the renormalization group may be relevant, that is whether the observable value of the CC can depend on time due to the quantum effects. At the first glance this question looks as something absurd. Usually, the low-energy effects of massive quantum fields manifest the quadratic decoupling at low energies [41]. Is it true that, despite the huge difference in the magnitude of the energy scales, the quantum effects can be relevant for the CC? Curiously, the answer is yes. In this section, we shall present the basic ideas of the approach of [8, 11] based on the notion of 'soft decoupling' of massive fields at low energies in the gravitational sector (see also [42]). One can find many important details in these papers and also in the last related developments in [12, 43, 44], in particular the implications of a running CC on the matter power spectrum [45].

Despite the calculations of decoupling for massive fields on curved background [46] were not successful in the CC sector, but the decoupling in other sectors of the vacuum action is of the standard quadratic form. Let us use a phenomenological approach and assume that the quadratic decoupling holds for a CC. In the present-day universe one can associate $\mu \equiv H$ [8].

From equation (2) remember that $\beta_\Lambda \sim m^4$, with m being the mass of a contributing quantum field. Then the quadratically suppressed expression is [8, 11]

$$H\frac{\mathrm{d}\Lambda}{\mathrm{d}H} = \beta_\Lambda = \sum_i c_i \frac{H^2}{m_i^2} \times m_i^4 = \frac{\sigma}{(4\pi)^2} M^2 H^2, \tag{11}$$

where M is an unknown mass parameter and $\sigma = \pm 1$ depending on whether fermions or bosons dominate in the particle spectrum. Assuming $M^2 = M_P^2$, we find $|\beta_\Lambda| \sim 10^{-47}$ GeV^4, which is close to the existing supernovae and CMB data for the vacuum energy density. Therefore, the renormalization group may, in principle, explain a smooth variation of the vacuum energy without introducing special entities like quintessence.

Two cosmological models with running CC have been developed in [11, 12], based on the original RG framework of [8]. Further developments around these models have been presented in [43, 44] with interesting implications on the coincidence problem. The renormalization group equation (11) leads to $\Lambda = \Lambda_0 + \sigma M^2\left(H^2 - H_0^2\right)\big/(32\pi^2)$. Furthermore, there is the Friedmann equation $H^2 = (8\pi G/3)(\rho + \Lambda)$ and the conservation law, which can be chosen in different ways. In one of the possibilities [11] we can admit the energy exchange between the vacuum and matter sectors (see also [47]), so that we have $\dot{\Lambda} + \dot{\rho} + 3H\rho = 0$. The solution of this set of coupled equations is completely analytical and the effect of the running is parametrized by the dimensionless parameter $\nu = \sigma M^2/12\pi M_P^2$. When $\nu \to 0$ we recover the standard result for $\Lambda = \text{const}$. The value of $|\nu|$ has to satisfy the constraint $|\nu| \ll 1$, typically $|\nu| \lesssim 10^{-2}$, for otherwise there is a dominance of DE over radiation in the nucleosynthesis time [11]. It was only later recognized that the strongest constraint actually comes from the computation of the density perturbations [45], where the consistency with the LSS galaxies' distribution data requires $|\nu|$ to be, at most, 10^{-4} with the best fit corresponding to values smaller than 10^{-6}. Qualitatively similar results have been achieved earlier in the framework of analogous models and, more quantitatively, in modified quintessence models [48].

The second RG framework [12] does not permit the energy exchange between vacuum and matter sectors. This is a good point, because (a) the conservation law is nothing but a mathematical expression of covariance. We have no reason to think that the vacuum and matter effective actions are not separately covariant. (b) The energy exchange between vacuum and matter assumes the creation of particles and the creation of both massive and massless particles in the present-day universe meets obvious obstacles. The conservation law for the vacuum action with variable CC requires that the Newton constant becomes weakly dependent on the Hubble parameter. Investigation of density perturbations in this model is an open problem and should be explored soon.

8. Conclusions

We presented a short review of CC problems. The fine tuning of CC is a hierarchy problem due to the huge difference between the particle physics and cosmological scales. Different approaches for solving this problems have been developed. The anthropic considerations, together with the idea of multiple vacuum states coming from string theory, give an important hint about possible values of CC. The renormalization group, in the framework of quantum field theory in curved spacetime, indicates the possibility of slowly varying CC even at the cosmic low energy scale.

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